## Exercise 2.4.1

Use linear stability analysis to classify the fixed points of the following systems. If linear stability analysis fails because $f'(x^*) = 0$, use a graphical argument to decide the stability.

$$\dot{x} = x(1 - x)$$

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**Solution**

The fixed points occur where $\dot{x} = 0$.

$$x^*(1 - x^*) = 0$$

$$x^* = 0 \quad \text{or} \quad 1 - x^* = 0$$

$$x^* = 0 \quad \text{or} \quad x^* = 1$$

Use linear stability analysis to classify these points.

$$f(x) = x(1 - x)$$

$$= x - x^2$$

Differentiate $f(x)$.

$$f'(x) = 1 - 2x$$

As a result,

$$f'(0) = 1 > 0 \qquad \Rightarrow \qquad x^* = 0 \text{ is an unstable fixed point.}$$

$$f'(1) = -1 < 0 \qquad \Rightarrow \qquad x^* = 1 \text{ is a stable fixed point.}$$

The graph of $\dot{x}$ versus $x$ confirms these results.